Type Inference for Ratio Control Multiset-Based Systems*

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Outline. The paper presents a hierarchical nested system together with a multiset-based type system involving a ratio control of the resources. This ratio control keeps the resources between lower and upper thresholds. According to a typed semantics, each rule of the system can be applied only if the left-hand side term of the rule is well-typed. A type inference is defined for deducing the type of each term of such a system. Soundness and completeness results are proved for this type inference.

Ecological stoichiometry is the study of the balance of energy and multiple elements in ecological interactions [10]. From the stoichiometric point of view, both quantity and quality need to be implicitly or explicitly modelled in producer-consumer interactions. Simple phenomenological two-dimensional producer-consumer models are mathematically tractable by extensions of the standard theory of predator-prey interactions described by Lotka and Volterra. The Lotka-Volterra model extended with the ratio-dependent interaction is presented in a rather recent book [2]. Usually, the ratio of (two) elements reflects the quality [11]. However, the models described by differential equations [7], do not explicitly track the quantities neither in the producer-consumer nor in the environment. In this paper we provide a discrete approach and a new quantitative (multiset-based) type system involving a ratio control of the resources.

Multiset-based formalisms are motivated by quantitative evolutions of various systems. In many biological systems a reaction takes place only if certain ratios between given thresholds are fulfilled (e.g, in sodium/potassium pump [3] and ratio-dependent predator-prey systems [6]). Several multiset rewriting systems are used to describe the dynamics of systems which involve parallelism and concurrent access to resources. Petri nets [9] and membrane systems [8] represent good examples of the kind of multiset-based formalisms mentioned in this paper.

The current paper presents a more general and improved approach starting from the type system of [1] in which we emphasise the ratio control of resources between lower and upper thresholds. In this way, we are able to capture the quantitative aspects and abstract conditions associated with correct evolutions. We provide a “two steps” description of behaviours, where the first step describes reactions in an “untyped” setting, and the second rules out some evolutions by imposing certain ratio thresholds. This approach allows to treat separately different aspects of modelling: first what are the possible transitions, and then under which circumstances can they take place. Also it facilitates a better understanding of the evolution in complex quantitative systems with lower and upper thresholds. We prove soundness and completeness results, and show that each rule of the system can be applied only if the left-hand side term of the rule is well-typed.

Let T be a finite set of basic types ranged over by t. Each object a in O is classified with an element of T; Γ denotes this classification. In general, different objects a and b can have

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the same basic type t (e.g, a and b can be both ions). When there is no ambiguity, the type associated with an object a is denoted by t_a. For each ordered pair of basic types (t_1, t_2), the existence of one function is assumed: $min : T \times T \to (0, \infty) \cup \{\diamond\}$. This function indicates the minimum ratio between the number of objects of basic types t_1 and t_2 that can be present inside a component. We consider that the maximum ratio between t_1 and t_2, denoted by $max(t_1, t_2)$ could be determined using the relation $min(t_1, t_2) \cdot max(t_2, t_1) = 1$. According to this relation, it is enough to use only the min function. For example, by taking the constraints $min(t_a, t_b) = 3$ and $min(t_b, t_a) = 1/5$, the number of objects of basic type t_a is larger than the number of objects of basic type t_b with a coefficient between three and five. $min(t_1, t_2) = \diamond$ tells that this function is undefined for the pair of types (t_1, t_2). Biologically speaking, the ratio between the types t_1 and t_2 is either unknown, or can be ignored.

We provide a type inference algorithm for deducing the type of each term in the multiset framework, and prove the soundness and completeness results for this type inference. We provide a typed semantics, and prove that each rule of the system can be applied only if the left-hand side term of the rule is well-typed.

A formalism that is somehow related to the multiset framework with components considered in this paper is the calculus of looping sequences. An essential difference is that multiset framework with components use multisets to describe objects on components, while calculus of looping sequences terms use words as looping sequences. There are various type systems defined for calculus of looping sequences. Our approach is related to [4], where a type system and type inference for the calculus of looping sequences is defined based on the number of elements. However, our type system and type inference approach uses ratio thresholds instead of exact numbers of elements being able to type systems more complex than in [4], providing also more flexibility given by lower and upper thresholds.

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