A Dialectica-Like Approach to Tree Automata

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We propose a fibred monoidal closed category of alternating tree automata, based on a Dialectica-like approach to the fibred game model of [15].

Alternating tree automata are equivalent in expressive power to the Monadic Second-Order Logic on infinite trees (MSO), which subsumes most of the logics used in verification (see e.g. [5]). Acceptance of an input tree t by an automaton $\mathcal{A}$ can be described by a two-player game, where the *Proponent* P (also called *Automaton* or ∃loïse) tries to force the execution of the automaton on a successful path, while its *Opponent* O (∀belard) tries to find a failing path. Then $\mathcal{A}$ accepts t iff P has a winning strategy in this game. Alternating automata are easily closed under complement, and together with the translation of alternating automata to non-deterministic ones (the *Simulation Theorem* [12]), this provides a convenient decomposition of the translation of MSO formulas to automata (see e.g. [5]), implying the decidability of MSO [14]. This work shows that to some extent this decomposition can be reflected in the decomposition of intuitionistic logic in linear logic [4].

The fibred symmetric monoidal closed structure allows to organize tree automata in a deduction system for a first-order multiplicative linear logic. Our model, which is based on game semantics, provides following [15] a realizability interpretation of this system: From a derivation of say $\mathcal{A} \multimap \mathcal{B}$, we can compute a strategy σ witnessing the inclusion $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})$ (where $\mathcal{L}(\mathcal{A})$ is the set of trees accepted by $\mathcal{A}$), in the sense that for any input tree t and any strategy τ witnessing that $t \in \mathcal{L}(\mathcal{A})$, the strategy $t^*(\sigma) \circ \tau$ witnesses that $t \in \mathcal{L}(\mathcal{B})$ (here t^* is the substitution functor induced by t).

We use Gödel's *Dialectica* interpretation (see e.g. [1, 11]) in two related ways. First, transitions of alternating tree automata can be seen (following e.g. [12]) as being valued in a free distributive lattice, hence as being given by expressions in a $\vee\wedge$-form. Then, Dialectica, seen as a constructive notion of prenex $\exists\forall$-formulas, provides the transition function of the internal linear implication of our notion of tree automata. Second, our notion of morphism (issued from [15]) is based on *zig-zag* strategies, which can be represented by a Dialectica-like category (see e.g. [3, 7, 6]) based on the topos of trees (see e.g. [2]). This allows to conveniently describe the dependencies of strategies on tree directions, and to get a very simple fibred structure thanks to a variation of simple fibrations based on comonoid indexing (see e.g. [8]).

When restricting to parity automata, the winning conditions of games of the form $\mathcal{A}_1 \otimes \cdots \otimes \mathcal{A}_n \multimap \mathcal{B}$ are given by disjunctions of parity conditions (called *Rabin conditions*), and it is known that if P wins such a game, then he has a positional winning strategy [10, 9]. In this context, we show that a powerset operation translating an alternating automaton to an equivalent non-deterministic one satisfies the deduction rules of

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the '!' modality of linear logic. Unfortunately, positional strategies do not compose, but we still get a deduction system for intuitionistic linear logic, which in particular gives deduction for minimal intuitionistic predicate logic via the Girard translation. Using a suitable negative translation based on the '?' modality, we can interpret proofs of minimal classical logic, and also get a weak form of completeness of our realizers w.r.t. language inclusion, in the sense that if $\mathcal{L}(\mathcal{A}) \subseteq \mathcal{L}(\mathcal{B})$, then from a regular O-strategy witnessing $\mathcal{L}(!\mathcal{A}) \cap \mathcal{L}(!(\mathcal{B}^{\perp})) = \emptyset$ (provided by an algorithm solving regular games on finite graphs, see e.g. [13]), we can build a winning P-strategy on $!\mathcal{A} \multimap (!(\mathcal{B}^{\perp}))^{\perp}$.

Details can be found in the unpublished draft [16].

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