Intersection Types for denotational semantics

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Intersection types can supply a tool for reasoning in a finitary way about the denotation of terms in models of lambda calculus. I will show how intersection type assignment systems can be tailored in such a way of describing denotational interpretation of terms in three very general classes of λ-models, namely Scott models, based on domains of continuous functions, Girard models, based on the domains of stable functions, and relational models, based on the category of sets and relations, recently defined by Ehrhard, Bucciarelli and Manzonetto, starting from an idea of Selinger. Each one of the three classes is described by a parametric type assignment system, when a particular model is grasped by a suitable instance of the parameter. The three parametric type assignment systems share a similar structure, but they differ not only in the parametric rules, but also in the properties of the intersection and in the structural rules. Namely, intersection needs to be idempotent to describe functional semantics, but not idempotent to describe relational semantics, weakening is necessary for the continuous semantics, while it is unsound in both the stable and relational semantics. Despite the differences between the various approaches, in each case the problem asking if two closed terms are equal in a model reduces to that one of asking if they can be assigned the same set of types in a given type assignment system. There are finitary techniques for solving the last problem: so intersection types supply a tool for proving semantical properties of terms, which can be applied in a uniform way to different kinds of models.

References

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